## Indian Statistical Institute, Bangalore Centre

Algebraic Topology (B. Math (Hons.) III year)

MID-SEM (30 MARKS)

**Question 1.** The wedge sum  $M_1 \vee M_2$  of two triangulable topological spaces  $M_1$  and  $M_2$  along points  $x_i \in M_i$  is the quotient space of the disjoint union of  $M_1$  and  $M_2$  by the identification  $x_1 \sim x_2$ :

$$M_1 \vee M_2 = (M_1 \amalg M_2) / \sim,$$

where  $\sim$  is the equivalence closure of the relation  $\{(x_1, x_2)\}$ .

- (a) Compute the Euler characteristic of  $M_1 \vee M_2$  in terms of Euler characteristics of  $M_1$  and  $M_2$ . (2 Marks)
- (b) Consider the wedge sum  $T \lor K$  of a torus T and a Klein bottle K. Compute the homology groups of  $T \lor K$ . (8 Marks)

**Question 2.** Does there exist a triangulation of the Klein bottle with 13 2–simplexes? Justify your answer. (4 Marks)

Question 3. Does there exist polyhedras |K| and |L| for which  $H_i(K)$  and  $H_i(L)$  are isomorphic for each value of i, but |K| and |L| are not homeomorphic? Justify your answer. (5 Marks)

Question 4. Let X and Y be topological spaces. Furthermore, assume that Y is contractible. Prove that if  $f: X \to Y$  is a continuous map, then f is null-homotopic. (3 Marks)

**Question 5.** Let X be a topological space. Let  $B^{n+1}$  be a closed (n + 1)dimensional ball in  $\mathbb{R}^{n+1}$  and  $S^n (= \partial B^{n+1})$  be the sphere of dimension n.

Suppose that  $f: S^n \to X$  is a continuous map. Then prove that the map f is null-homotopic if and only if f has a continuous extension  $F: B^{n+1} \to X$ . (5 Marks)

**Question 6.** Two spaces X and Y are said to be homotopic equivalent (or to have the same homotopic type) if there are maps  $f : X \to Y$  and  $g: Y \to X$  such that  $g \circ f$  is homotopic to the identity map  $id_X$  of X and  $f \circ g$  is homotopic to the identity map  $id_Y$  of Y.

Does the homotopy type of  $\mathbb{R}^3$  minus z-axis and the circle  $S^1$  are the same? Justify your answer. (3 Marks)

## END OF PAPER